

WITTGENSTEIN, METAPHYSICS, AND THE OPERATIONAL
APPROACH TO THE STUDY OF LANGUAGES

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RESUMEN

Este artículo defiende la tesis de que las doctrinas metafísicas, debido al alto grado de generalidad y abstracción de su discurso, pueden ser reinterpretadas a la luz de los hallazgos teóricos de la lingüística operacional. Ilustra esta tesis mediante un estudio limitado, pero representativo, del *Tractatus* de Wittgenstein, cuya crítica del uso filosófico y lógico del concepto de objeto basada en la introducción del concepto de "concepto formal" se ve iluminada y sistematizada mediante la aplicación de la teoría de la aprehensión lingüística de objetos a la explicación de las diversas variables utilizadas en lenguajes formales de diferentes niveles de abstracción.

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WITTGENSTEIN, METAPHYSICS, AND THE OPERATIONAL APPROACH TO
THE STUDY OF LANGUAGES

1. *Introduction*

Languages enable us to talk about objects. To be able to talk about objects implies to be able to first apprehend those same objects. At first sight, such dicta might seem to belong to those a priori pontifications that cannot be dismissed as false simply because they are too trivial. That they are much more, and especially that it is empirically respectable and rewarding to try and approach the study of language and languages by asking how they do apprehend objects, is the merit of a ten-year teamwork of the UNITYP research group, as set down in the recently completed trilogy on the APPREHENSION of objects.¹

But what does APPREHENSION amount to? and what are the objects to be apprehended? According to the operational approach of UNITYP, these two questions cannot be answered separately, because to be (an object) is to be apprehended (as an object). It is important that the word "object" keeps here the same all-embracing generality which it has in everyday parlance. This chair is as much an object as the reduction of nuclear weapons is one. Or to speak linguistically, the English word *chair* is as good an instrument to apprehend an object as is the English NP *reduction of nuclear weapons*. But there are surely differences? Yes, indeed there are. And to explicate them, the UNITYP group has devised a powerful theoretical construct—the dimension of APPREHENSION.

We insist on using capitals because the word "apprehension", in its ordinary use, is a label for something that still has to be understood and thus not an explanation but at best a promise of an explanation; the capitals are there to suggest that a theory is being offered to support the use of the word. According to this theory—the UNITYP theory—APPREHENSION is

a very complex operation that is inter alia different from and prior to what is variously called "denotation", "reference", "extension", and "designation" (all of whom, conveniently re-interpreted, belong to the dimension of DETERMINATION). To call APPREHENSION complex is to say that it itself consists of several operations which interact with each other along a continuum of linguistic "techniques". These "techniques" (or "subdimensions") are related, but not identical, to a group of categories that have been postulated in the descriptions offered for very different languages, e.g. abstract nouns, mass nouns, collectives, singulatives, classificatory verbs, numeral classifiers, noun class, gender, number, etc.

On the other hand, to speak of a continuum of techniques is to suggest that each technique has got a hard core, manifested prototypically by certain constructions and regularities, and that other constructions and regularities are more marginal; so much so that a kind of transference can take place between neighbouring techniques. Thus, an abstract noun can be "projected" into one of its arguments (positions, slots, empty places); this will result in a loss of "saturability" (the possibility for more or less oblique NPs to occupy the empty places of the original predicate is reduced); the noun becomes therefore marginal to the maximally predicative technique of ABSTRACTION and can be taken over by the neighbouring technique of COLLECTION. For instance, the Latin word *humanitas*, in keeping with the predicative power it inherits from the adjective *humanus*, can enter into a construction like *humanitas populi Romani* or *humanitas Ciceronis*; but if it is "projected" into its own subject position, it develops into a collective noun that cannot be thus saturated; as against this, it can now enter into constructions like *pars humanitatis*, by which a subclass is extracted from a given class. The word *humanitas* has then, by the said "projection", been transferred from ABSTRACTION onto COLLECTION.

Such a state of affairs can be expressed both statically and dynamically. Statically, we can say that *humanitas* as an abstract noun represents a different kind of object from *humanitas* as a collective noun. It is the traditional way of talking both in philosophy and in linguistics. In philosophy, it is manifested in the distinction between relations (or complexes or propositional functions) and classes. In linguistics it can lead to the distinction of two lexemes, *humanitas*₁ and *humanitas*₂. The linguists and philosophers of UNITYP prefer a more dynamic talk; we would say that the operations involved in the constructions *humanitas Ciceronis* and *pars humanitatis* are different; and that the objects in question are constituted—apprehended—as a result of those same operations. To speak of different kinds of objects (and of different lexemes) does not really help, because the objects are nothing without the operations.²

Hence, talk of objects in general is only possible on an operational basis. Now, philosophers, from the oldest times to the most recent ones, have always been willing to talk of objects in general. In the following, I will try to throw light on the operational approach to objects by going back to some of the things philosophers have said about them. I cannot offer more than a fragment of the story; but I offer it in the hope that it will prove useful and stimulating to both philosophers and linguists.

2. The word object in philosophy

An indispensable tool in order to speak about objects in general is, precisely, a word like English *object*. Its incredible range is what makes it so useful—for everyday life as well as for metaphysical speculation. Of course, the latter contains tokens of the word that would sound strange, if not eery, to the proverbial man in the street. This is one of the reasons why its metaphysical use has time and again been challenged by rebellious voices both within and without philosophy. In our

century, we associate the most vigorous among them with controversial labels like "logical positivism" and "the Vienna School". Apart from controversy, there is no denying the fact that Wittgenstein's immensely influential *Logisch-philosophische Abhandlung* (re-baptized by G.E. Moore as *Tractatus logico-philosophicus*) belongs to the anti-metaphysical frame of mind I was referring to. Now, the *Tractatus* is especially important for us because the attack it mounts upon the German word *Gegenstand* ('object') and similar words like *Ding* or *Sache* ('thing', 'matter') goes together with a very deep reflection not only about the words themselves but—more importantly—about the constitution of objects through language, i.e. about APPREHENSION.

The decisive step in Wittgenstein's considerations is the idea that the word *object* denotes a "formal concept".³ Before explaining what a formal concept is, I need to explain what a concept is in the first place, given that the word has been so variously used. Wittgenstein got the word *concept* (i.e. *Begriff*) from his acknowledged master, Gottlob Frege, who wrote:

Der Begriff — wie ich das Wort verstehe — ist prädikativ. [Er ist nämlich Bedeutung eines grammatischen Prädikats.] Ein Gegenstandsname hingegen, ein Eigennamen ist durchaus unfähig, als grammatisches Prädikat gebraucht zu werden. (Frege 1892:193. The original has the words within square brackets as a footnote to the main text.)

Frege opposes here the proper nouns to the verbs, adjectives, and common nouns of traditional (Latin-based) grammar. By the way, Frege brings the latter three categories together under the label *Begriffswort* (literally 'concept-word') which is strongly reminiscent of the now old-fashioned vocabulary of nineteenth-century German grammarians and schoolmasters (cf. *Handlungswort* for 'verb', *Beziehungswort* for 'preposition', etc.). Proper nouns ("object names") are not "predicative", but "concept words" are. That is why one often feels tempted to translate Frege's *Begriff* by English *predicate* (see below).

Now, to the possible objection that proper nouns may occur as predicate nominals in the same manner as common nouns or adjectives, e.g. in the environment *NP is ___*, and therefore are "predicative", Frege replies that the verb "to be" is ambiguous. When it appears in connexion with adjectives and common nouns,

dient es als Kopula, als blosses Formwort der Aussage. Als solches kann es zuweilen durch die blosser Personalendung vertreten werden. Man vergleiche z.B. "dieses Blatt ist grün" und "dieses Blatt grünt". Wir sagen dann, dass etwas unter einen Begriff falle, und das grammatische Prädikat bedeutet dabei diesen Begriff. (Frege 1892:194.)

The reader is asked to keep in mind the expression "to fall under a concept" because Wittgenstein will use it, too. Now, when the verb "to be" appears in connexion with proper nouns, it is used

wie in der Arithmetik das Gleichheitszeichen..., um eine Gleichung auszusprechen. Im Satze "der Morgenstern ist die Venus" haben wir zwei Eigennamen "Morgenstern" und "Venus" für denselben Gegenstand. In dem Satze "der Morgenstern ist ein Planet" haben wir einen Eigennamen: "der Morgenstern" und ein Begriffswort: "ein Planet". Sprachlich zwar ist nichts geschehen, als dass die "Venus" ersetzt ist durch "ein Planet"; aber sachlich ist die Beziehung eine ganz andere geworden. Eine Gleichung ist umkehrbar; das Fallen eines Gegenstandes unter einen Begriff ist eine nicht umkehrbare Beziehung. Das "ist" im Satze "der Morgenstern ist die Venus" ist offenbar nicht die blosser Kopula, sondern auch inhaltlich ein wesentlicher Teil des Prädikats, so dass in den Worten: "die Venus" nicht das ganze Prädikat enthalten ist. Man könnte dafür sagen: "der Morgenstern ist nichts anderes als die Venus", und hier haben wir, was vorhin in dem einfachen "ist" lag, in vier Worte auseinandergelagt, und in "ist nichts anderes als" ist nun "ist" wirklich nur noch die Kopula. Was hier ausgesagt wird, ist also nicht *die Venus*, sondern *nichts anderes als die Venus*. Diese Worte bedeuten einen Begriff, unter den freilich nur ein einziger Gegenstand fällt. Aber ein solcher Begriff muss immer noch von dem Gegenstande unterschieden werden. (Frege 1892:194.)

From the viewpoint of linguistic analysis, Frege's exposition is certainly not above criticism; but we only want here to get acquainted with his terminology.

All right then, the word *object* is not a proper noun; hence

it "denotes" a concept, either by itself or in association with other constituents of a well-formed noun phrase. Now, according to Wittgenstein, the sensible use of the noun *object* must conform to certain restrictions which have not always been respected by philosophers and logicians. It is not an ordinary common noun; ordinary common nouns denote genuine concepts, but the noun *object* does not—it just denotes a formal concept. The adjective *formal*, as used by Wittgenstein, seems to belong to that small subclass of adjectives whose prototype is the adjective *false*: as a false friend is not a friend, thus a formal concept is not a concept. Some readers will be immediately reminded of Aristotle's analogous dictum: "Being is not a genus". And indeed there is a connexion between both positions; but the difference of contexts is such that it would not be sensible to pursue the comparison further. Anyway, a future comparison would presuppose an exposition of the reasons for Wittgenstein's opinion. Let us then explain them briefly.

The foundational paradoxes—among which the one that bears Russell's name is the most famous—seemed all to imply the possibility of a sentence to speak about itself. Thus, to eliminate paradoxes it was necessary to avoid this possibility; but in order to repair the damage done by such a prohibition Russell had devised a number of axioms, rules and theories which Wittgenstein found self-defeating. One of Russell's devices is known as the "axiom of infinity", which in its most popular form says: *There are infinitely many objects in the world*. This Wittgenstein considers plain nonsense, apparently because you cannot count objects the way you can count, say, books; or, phrased more linguistically, the word *object* can enter a count NP only if accompanied by certain other determiners. A long quotation is here necessary:⁴

So ist der variable Name "x" das eigentliche Zeichen des Scheinbegriffes *Gegenstand*. Wo immer das Wort "Gegenstand" ("Ding", "Sache", etc.) richtig gebraucht wird, wird es in der Begriffsschrift durch den variablen Namen ausgedrückt. Zum Beispiel in

dem Satz: "Es gibt 2 Gegenstände, welche...", durch " $(\exists x, y) \dots$ ". Wo immer es anders, also als eigentliches Begriffswort, gebraucht wird, entstehen unsinnige Scheinsätze. So kann man z.B. nicht sagen: "Es gibt Gegenstände", wie man etwa sagt: "Es gibt Bücher". Und ebenso wenig: "Es gibt 100 Gegenstände", oder: "Es gibt Aleph-Null Gegenstände" [i.e. "There are as many objects as natural numbers" or "There are infinitely many objects"]. Und es ist unsinnig, von der *Anzahl aller Gegenstände* zu sprechen.

Dasselbe gilt von den Worten "Komplex", "Tatsache", "Funktion", "Zahl", etc. Sie alle bezeichnen formale Begriffe und werden in der Begriffsschrift durch Variable, nicht durch Funktionen oder Klassen dargestellt. (Wie Frege und Russell glaubten.) Ausdrücke wie: "1 ist eine Zahl", "Es gibt nur Eine Null" und alle ähnlichen sind unsinnig. (Es ist ebenso unsinnig zu sagen: "Es gibt nur Eine 1", als es unsinnig wäre, zu sagen: "2+2 ist um 3 Uhr gleich 4".)

A part of my task in this paper will be to explain the meaning of this passage. I must begin with relatively simple, terminological matters.

First, Wittgenstein is clearly alluding to the way Frege and Russell talk about the symbolism they invented (in imitation of that of infinitesimal calculus). This symbolism was called by Frege a *Begriffsschrift*, which is the expression also used by Wittgenstein and refers to Frege's above-mentioned terms *Begriff*, *Begriffswort*, etc.; Russell spoke rather of the "logic of relations", thus reflecting his transformation of Peano's symbolism. And when Hilbert introduced the expression "predicate calculus", he was just following Frege's tradition; and when Quine began to talk of the "logic of quantification", he was following Russell's.

Second, in this symbolism "concepts" or "predicates" were represented partly as "functions" (the so-called "propositional functions") and partly as "classes"—both opposed to "objects" or "individuals". (Russell liked to speak of "universals" vs. "particulars".) The convention was adopted that different types of letters were used to signify both. Thus, the letter *f* (the mathematical symbol for a function) was used to signify a predicate and the letter *x* (the mathematical symbol for the argument of a function) was used to signify an object;

and therefore a complex symbol like $f(x)$ was used to signify a proposition. When Wittgenstein says that formal concepts should be represented by variables not by functions or classes, he means that the symbols for formal concepts cannot be used to predicate.⁵ This would clearly eliminate sentences like *Concept is a concept* as not well-formed (and therefore nonsensical).⁶

Third, words like *fact* and especially *complex* were used in the writings of Russell as a result of discussion with Meinong and Moore.⁷ In a somewhat simplified manner, one can say that a proposition might be considered either as asserted or as merely presumed, imagined, contemplated or supposed; a fact would then be the referent or denotatum of an asserted proposition (a statement) and a complex that of a non-asserted one. Thus, a conditional sentence signifies a fact, but any of its two parts (the antecedens and the consequens) would only signify a complex.⁸ Sometimes the complex was represented linguistically not by an articulated sentence but by an abstract noun with oblique arguments. We shall come back to that.

Fourth, Wittgenstein characterizes x not as a variable but as a "variable name", which is against contemporary use. We are indeed used to consider the word *variable* as a substantive rather than as an adjective; but this has not always been the case; nineteenth-century mathematicians still spoke of the "variable number". Frege criticized this usage with arguments which were as powerful as they were mordant (e.g. in Frege 1904). Wittgenstein was obviously not impressed, and to the old "variable number" (6.022) he added the "variable proposition" (3.315) and, as we have just seen, the "variable name" (cf. also 3.314).⁹ The reason, I think, is that for Wittgenstein every variable denotes a formal concept (cf. also 4.1271). And indeed there seems to be no limit to the number of formal concepts.¹⁰

3. Variables and objects

From a linguistic point of view, the most interesting point of the passage quoted is that Wittgenstein breaks a long tradition of metaphysical thinking not so much by renouncing to talk about objects in general in order to talk about words like *object* (earlier philosophers had done either both things or at least one) but by going from the use of such words to the apprehension of objects through language. The linguistic instrument in question, so he tells us, is the "variable name". Now, the "variable name", the letter x , is one of several small-case letters used by mathematicians to indicate numbers (or points), the two other most popular letters being y and z . The fact that Wittgenstein does not allude to these letters, but only mentions x , will occupy us in a moment. The point is that all these small-case letters are not used by mathematicians to indicate any old objects but just numbers. As I said before, the predicate calculus is a symbolism constructed in imitation of the infinitesimal calculus which is only about numbers (more precisely about real numbers); the former is thus a kind of extension of the latter in that the variables seem now to refer to all kinds of objects; that is what the celebrated all-embracing generality of symbolic logic (*la physique d'un objet quelconque*, in Gosset's phrase) is about and the ultimate reason why symbolic logic was thought by Wittgenstein's teachers to be able to support the whole edifice of mathematics.

However, Wittgenstein was very much worried about this whole matter, not least because of the enormous differences between the logical symbolism and the natural languages which indeed are about any object. In this connexion, the notebooks he wrote in preparation for the *Tractatus* contain the following instructive considerations:

... wir wenden ja die Mathematik, und zwar mit bestem Erfolge, auf die gewöhnlichen Sätze, nämlich auf die der Physik, an!! Aber wie merkwürdig: in den bekannten Lehrsätzen der mathematischen Physik erscheinen weder Dinge noch Funktionen noch Relationen [i.e. ordinary predicates] noch sonst logische Gegenstandsformen!! Statt der Dinge haben wir da Zahlen, und die Funktionen und Relationen sind durchweg rein mathematisch!! Aber es ist doch Tatsache, dass diese Sätze auf die solide Wirklichkeit angewandt werden. Die Variablen in jenen Lehrsätzen stehen durchaus nicht — wie man häufig sagt — für Längen, Gewichte, Zeiträume etc., sondern sie stehen einfach für Zahlen und weiter nichts. (Von Wright/Anscombe eds. 1979:66-67.)

A solution to the puzzle would be to accept that variables which play the same syntactic role as the letter x , i.e. which may be arguments of functions and terms of relations, serve to apprehend objects of any kind whatever. To which "kind" a given object belongs might depend on the kind of function or relation it is an argument or a term of. This is most decidedly the position adopted by UNITYP; and I think there is evidence, both direct and indirect, in Wittgenstein's writings which confirms that something like this was also his view—although he was, on the other hand, very much against talking of "kinds" of objects.

The most direct and general type of evidence can be drawn from passages like the following, in which Wittgenstein was protesting against the confusion of language levels (what was later to be called language vs. metalanguage):

Der formale Begriff ist mit einem Gegenstand, der unter ihn fällt, bereits gegeben. Man kann also nicht Gegenstände eines formalen Begriffes und *den* formalen Begriff selbst als Grundbegriffe einführen. Man kann also z.B. nicht den Begriff der Funktion, und auch spezielle Funktionen (wie Russell) als Grundbegriffe einführen; oder den Begriff der Zahl und bestimmte Zahlen. (4.12721; cf. also 4.127.)

I strongly suspect that the word *object* is so to speak shorthand for the complex expression *object that falls under a concept*. In other words, an object is always relative to a concept. The concept of an object is thus the most general formal concept and the letter x the widest and emptiest of variables. There are here clear parallels to older philoso-

phical usage, as I pointed out elsewhere (Leal 1983). The point is that the word *object* as a covertly relational term indicates the converse relation to that of predication; if we tried to express it verbally, we would come to something like Tarski's relation of satisfaction (an object would be a "satisfactor"). Thus *object* and *concept* are complementary terms—a view that has been defended by UNITYP members (cf. Seiler 1978)—and the letter x a very apt expression for such emptiness (I shall come back to this in the next section). But if it is so, then any variable, and not only x , stands for a formal concept and indicates a "kind" of object.

This interpretation is reinforced by some notebook remarks:

Ein Komplex ist eben ein Ding! (Von Wright/Anscombe eds. 1979: 49.)

Das, was uns a priori gegeben scheint, ist der Begriff: *Dieses*. — Identisch mit dem Begriff des *Gegenstands*. Auch Relationen und Eigenschaften etc. sind *Gegenstände*. (Ibid. 61.)

Such passages constitute a clear evidence, although the limits of its application may be discussed. The context in which the quoted remarks appear is Wittgenstein's effort to analyse the concept of an object and especially of a simple object. All arguments used by Russell about particulars, sense-data and the like are there subjected to a thorough scrutiny and critique, and it becomes clear that for Wittgenstein an object is that which we have a "name" for. This, however, depends upon the language or symbolism used and upon our needs. And that means that numbers, classes, and so on may be handled as objects as soon as we introduce "names" for them. The distance from APPREHENSION is not great now.

4. Quantification and identity

The most decisive evidence for the proposed interpretation, however, is the fact that the long passage quoted in Sec. 2 (i.e. 4.1272) binds quantification, and especially numeral

quantification ("there are n objects such that..."), together with objecthood. Now quantification can be applied to any variable. Objects are there to be counted; concepts are not. But if concepts are reified (by the technique of ABSTRACTION), then they can be counted, too. And there is more.

Frege was very much pre-occupied about the status of the mathematical identity sign. Sentences of the form " $a = a$ " were all true for him but also all trivial, whereas sentences of the form " $a = b$ " were either definitions (and had therefore no epistemological value at all) or else possibly true sentences (and often represented very valuable knowledge indeed). In order to show the difference between all these kinds of sentence, he devised his famous distinction between the sense and the reference of a given symbol. I cannot go into the details of this distinction or of the semantic theory it was embedded in.¹¹ But Wittgenstein saw the whole question in a very different light. He says that sentences of the form " $a = b$ " could only be definitions—otherwise they would be simply absurd.¹² Now, the whole discussion would be linguistically irrelevant if it were not for the symbolic consequence Wittgenstein drew from this position:

Gleichheit des Gegenstandes drücke ich durch Gleichheit des Zeichens aus, und nicht mit Hilfe eines Gleichheitszeichens, Verschiedenheit der Gegenstände durch Verschiedenheit der Zeichen. (5.53)

For instance:

Ich schreibe also nicht " $f(a,b).a = b$ ", sondern " $f(a,a)$ " (oder " $f(b,b)$ "). Und nicht " $f(a,b).-a = b$ ", sondern " $f(a,b)$ ". (5.531)

This was Wittgenstein's most serious symbolic proposal, as is shown by his later insistence on it (cf. e.g. Ambrose 1979: 146ff., 207f.). I do not think it is linguistically correct; and, inasmuch as it seems to break the type of generality introduced by variables in mathematical reasoning, I do not think it is very sound, either, from the viewpoint of the philosophy of mathematics (see next paragraph). But we are not

discussing here the use of the identity sign. What fascinates me is the operational aspect. It has been said that repetition of a variable along a logical "sentence" is related to grammatical agreement (cf. Lehmann 1982:233f.; Iturrioz 1986:34) and thus an operation of APPREHENSION: but the interesting thing is how well aware of the connexion between repetition and objecthood did Wittgenstein seem to be. He says in effect that the only way to ensure "equality of the object" must be through "equality of the sign"; this is astonishingly near to the UNITYP concept of an operational program of "object constancy" as one of the basic operations of APPREHENSION. Wittgenstein saw clearly that an assertion of equality is not a substitute, although his wanting to eliminate the equality sign altogether may be the wrong conclusion.

Now, the interesting question that arises here is whether variables constitute a new technique of APPREHENSION, different both from AGREEMENT, from NUMERAL CLASSIFICATION and from NAMEGIVING. According to the only linguistic analysis of variables so far available (Iturrioz 1986a:39; cf. the diagram in Iturrioz 1986b:303), it does. To this highly original study, I would like to add some brief remarks as to the difference between ordinary variables and numeral classifiers which might show where Wittgenstein went wrong. In natural language possessing numeral classifiers nouns do not ordinarily represent "quanta", so that an operation of "quantization" is required to make counting of objects possible (this "quantization" is a form of "individuation" or "objectification"). Now, such non-quantized nouns usually go together with one particular classifier; in the few cases where a classifier shift is possible, the concomitant predication is such that a different (kind of) object is apprehended. This does not seem to be the case in mathematics; the shift from x to y does not result in a shift of (kind of) object. It is the very same (kind of) object, e.g. a number.¹³ The traditional use of variables implies (pace Wittgenstein) that two diffe-

rent letters may refer to the same number. Thus the proof of the theorem $a(b + c) = ab + ac$ covers the particular case $a(a + b) = a^2 + ab$ without having to prove the latter by itself. In point of fact, Wittgenstein's variables are much more similar to certain dummy proper names like the *John Doe* and *Richard Roe* of law, or even the less technical *Jones* and *Smith* of everyday language and conversation (cf. Spanish *Fulano*, *Zutano*, *Mengano*, *Perengano*) than they are to traditional variables, in that they seem to preclude identity in the same context. It is worth mentioning here that the informal use of variables outside of mathematics invariably shows this discernibility—a fact that probably hangs together with the peculiar kind of thinking mathematical thinking is; but a more thorough study would be required before any progress can be made in the understanding of variables in their different uses. One among many important questions in this connexion would be, Why does the ordinary working mathematician seldom, if at all, use explicit quantification? Its absence from nonmathematical contexts is not surprising, but why did we have to wait for logicians to get it? I suspect all this depends upon the kind of sign an ordinary mathematical variable is.¹⁴

Now, besides Wittgenstein's implicit characterization of variables which thus manifests itself in his remarks on quantification and identity, he has also proposed new representations for them. We must turn our attention to them.

5. Objects and variables again

With the possible exception of identity, which he wanted to keep only for definitions, Wittgenstein did not make any symbolic proposal but for the sake of enlightenment. He seems to have had an idea about the use of calculi which is not dissimilar to the methodology proposed by his contemporary, the linguist Georg von der Gabelentz. The latter thought that "languages explain each other"; this is of course literally false because it is not language (any language) but

theory which is explanatory; it is a slogan which, if we spell it out, means that any linguistic theory worth its salt must be based on wide typological comparison. Wittgenstein, on the other hand, saw the point of designing a calculus in the light it throws on other calculi. Thus, speaking of his own version of propositional logic, the so-called truth table method, he said:¹⁵

Now who uses the calculus of T and F? I would say it has no use. Taken as a calculus, it is dull and useless, and so is Russell's calculus. But it has a justification which may not hold for other logics. The point of the T-F calculus is to afford a translation of Russell's calculus and its application. A calculus is of no value unless it makes one clearer about another. (Ambrose 1979:139.)

This is the real meaning of what was to be called "semantics" by later logicians. If there is a difference, it lies rather in the clarity of mind Wittgenstein shows. Compared to him, the logicians, philosophers and linguists who hope that a semantic calculus can substitute semantic theory are completely muddle-headed (see the above remarks about von Gabelentz's slogan). Any language, whether natural or formal, needs explanation; this explanation must certainly be couched in a language but it is not identical with it. A critical reader might wish to remark that Wittgenstein did not think much of explanation. Indeed, this has been expressed in the *Tractatus* quite often; and a quotation from a later writing confirms and reinforces this position:

We can't build up language. No explanation can go beyond symbolism. To understand an explanation we must understand symbolism... In translating you have nothing but grammar to guide you. You can't describe the application of language: you can't justify language. (Lee 1980:114f.)

In that point, as in others, Wittgenstein was wrong. One of the reasons why he was so skeptical about explaining language is his somewhat primitive idea of linguistic operations. When all is said and done, Wittgenstein's famous operations, as mentioned both in the *Tractatus* and in the earlier notebooks,

are nothing but distributional rules, which are certainly necessary for linguistic theory but belong to the lowest descriptive level (cf. Seiler 1982).

Indeed, Wittgenstein conceives an operation as a recursive procedure—similar to those of logical calculi and generative "grammars"—by which all expressions and sentences of a natural or artificial language are constructed. Thus, numerical expressions constitute a so-called "series of forms" which is defined by an operation of addition; for instance, integers have the "general form" expressed by the curious symbol $[0, \xi, \xi + 1]$. The whole series of the integers is contained in this symbol, because the first members of the series is zero (0), whereas any other member is the result of adding one to a given member of the series ($\xi + 1$, where the Greek letter ξ represents an arbitrary integer). Wittgenstein uses this kind of complex symbol again and again to illustrate several points; the structure is always the same: a triad of symbols of which the first represents always the "start" and the other two the operation as applied to any member—including and beginning with the "start" (cf. 5.2522, 6, 6.01, 6.03).

It is not quite clear whether Wittgenstein ever thought of these symbols as capable of forming part of a new calculus; but he refers to them as "variables"—a terminology which invites some questions. To see what I mean, consider the most general passage:

Die fortgesetzte Anwendung einer Operation auf ihr eigenes Resultat nenne ich ihre successive Anwendung ("O'O'O'a" ist das Resultat der dreimaligen Anwendung von "O'ξ" auf "a")...

Das allgemeine Glied einer Formenreihe a, O'a, O'O'a, ... schreibe ich daher so: "[a, x, O'x]". Dieser Klammerausdruck ist eine Variable. Das erste Glied des Klammerausdruckes ist der Anfang der Formenreihe, das zweite die Form eines beliebigen Gliedes x der Reihe und das dritte die Form desjenigen Gliedes der Reihe, welches auf x unmittelbar folgt. (5.2521-2522)

Now, if the complex symbol $[a, x, O'x]$ is a variable, so is its second part, the letter x . Which is the difference? Or to take Wittgenstein's own linguistic description over, which

is the difference between "the general member" of a series and "the form of any member" of the same series? I want to suggest it is a difference in APPREHENSION: the complex symbol is an analogue of what, in a natural language, we would call an abstract noun, whereas the single letter is more like a common noun (with the necessary reservations implied by what I said before).¹⁶ The objects apprehended by both symbols are not exactly identical, or rather they are not apprehended in the same way. In the case of integers, I would like to insist on the condensation of information that takes place when we go from the operation of counting (which is described in Wittgenstein's complex symbol) to the numbers themselves.¹⁷

To bring my point home I would like to recall the famous translation of the logical connective "⊃" (or "→") as "implies" in the early writings of Russell and other mathematical logicians. There has been an endless discussion about this translation.¹⁸ It was clear to everyone that if you translate $p \supset q$ as p implies q , then p and q are nouns. Some people went so far as to say that in such a case one had to write the letters within inverted commata: ' p ' implies ' q ', under the convention that inverted commata are a kind of quotative particle. A whole literature about inverted commata and quotation resulted. But a simpler solution is possible, viz. that p and q are similar to abstract nouns, the product of a nominalization of underlying predicates; and the symbol "⊃" is analogous to one of those few verbs that can accompany abstract nouns and thereby create a new universe of discourse within which it is possible to speak about propositional contents as objects. The second volume of Iturrioz (forthcoming) contains a powerful theory of ABSTRACTION in this sense which is based upon several different natural languages. (Lest the reader may misrepresent the above remarks as asserting that propositional letters are abstract nouns, I wrote a brief appendix in which I explain my view more fully; see below.)

I suspect that one reason why neither Wittgenstein nor anyone else took that step is that nobody could think of a verb

that could correspond to the logical connectives of "conjunction" (symbolized by "·", "∧" or "∧") and "disjunction" ("∨"). On the one hand, it was easy to identify them with the conjunctions "and", "or" of traditional grammars (although there was a little difficulty about the latter being supposedly ambiguous); on the other hand, the conjunction "if" could, albeit not without some rigmaroles, be a substitute for "implies"—so there was no need for verbs. We have here a classical case of influence of (natural) language on (logical) thought; had our logicians spoken a language like Dyirbal which possesses a verb to indicate "disjunction" or one of those African languages in which the conjunction "and" has evolved from a verb, then a different logic—or at least a different philosophy of logic and a different philosophy of language—might have ensued.¹⁹ As to negation, the incredible variety of its manifestations in natural languages include not only conjunctions (*but*) and prepositions (*without*) but also adjectives (*different*) and verbs (*lack*). Moreover, the "negative" verbs of English are, as far as I can see, eminently compatible with abstract nouns.

Now, although Wittgenstein thought that anything, if we are pleased to introduce appropriate symbols, can be apprehended as an object, he seemed to stop short before the logical connectives themselves—they are most emphatically said not to be relations (5.42, 5.461) and therefore not to be objects (5.4, 5.44).²⁰ I suppose this is because he could not think of a formal theory of connectives. However, his own informal theory contradicts him inasmuch as it might be formalized; moreover, insofar as he thought of connectives as truth tables, some of the symbols of his *Tractatus* are the beginning of a formal theory of logical connectives (cf. 4.27, 4.42, 4.45); in fact, his symbols for operations (\circ , as above, or Ω in 6.01) are variables ranging inter alia over logical connectives and hence even more general!

Of course, the letters p, q, r, \dots are analogues of abstract nouns with a vengeance. They are more or less completely reduced in variability. They represent the pole of maximum generality with no possibility of individualization by saturation of empty places. In Russell a symbol like $p \cdot \exists_x q$ is only a notational variant of the more usual $(x) \cdot \phi x \equiv \psi x$; and the use of indexes one finds occasionally—in a formula like $(x)(p_x \supset q_x)$ —only confirms it. Another candidate for ABSTRACTION would be that curious Meinong-Russell construct, the "complex". Complexes, as I said above, were conceived of as propositions deprived of their assertivity. Frege would have considered them as real propositions, insofar as they could be negated; but Russell refers to them always by means of NPs whose head noun is a nominalized predicate. (Negation is in fact not necessarily an obstacle, as phrases like *nonproliferation of nuclear weapons* show.) Thus, expressions denoting "complexes" allow for so many combinations as the most flexible abstract nouns one can think of. But nobody, as far as I know, has elaborated any theory or calculus of "complexes"; the whole subject never abandoned the hot but unfertile area of philosophical debate. As to Wittgenstein, he was content with acknowledging the status of "complex" as a formal concept, never going beyond the lapidary assertion that any propositions about "complexes" can be reduced to simple sentences (2.0201), which is most certainly false with respect to abstract nouns.²¹

6. Predicates and kinds of objects

We have been speaking informally of "kinds" of objects. The consistently used "scare quotes" were inter alia intended as a warning that Wittgenstein abhorred the whole idea of kinds of objects. A passage from later lectures deserves to be quoted in full:

There are no logical concepts, such for example as "thing", "complex" or "number". Such terms are expressions for lo-

gical forms, not concepts.

Roughly speaking, a concept can be expressed as a propositional function: e.g. $\phi() = ()$ is a man. But we cannot say $\phi() = ()$ is a number. Such logical concepts are pseudo-concepts and cannot be predicated as ordinary concepts are. They are properly expressed by a variable together with the rules applying to it, the rules for obtaining its values. So I cannot write $(\exists x).x$ is a number or $(\exists x).x$ is a thing. If I use this notation I must write $(\exists x_{n,m,t,r}).\phi x$, meaning that there are certain variables to which specified rules apply: the pseudo-concept occurs inside the \exists bracket, the true concept outside. All apparent logical concepts are to be expressed by a variable plus the grammatical rules governing its use...

That there are thus no logical concepts explains why it is nonsensical to classify in philosophy and logic. In philosophy and logic there are no "kinds" of thing, term and so on. In the sciences we draw distinctions and classify, and we do so by means of propositions which are true of one kind of thing and false of another. This is just what we can't do in logic. (Lee 1980:10-11.)

His point becomes still clearer when he uses one of his favourite examples, viz. colour. Wittgenstein draws the two diagrams for colours, the famous Newtonian circle and a square, in which he places the four primary colours (red, blue, green and yellow) in relation to each other, pointing out that only the former representation—the circle—is adequate to the facts ("has the correct multiplicity", i.e. homology). Then he says:

"Primary colour" and "colour" are pseudo-concepts. It is nonsense to say "Red is a colour", and to say "There are four primary colours" is the same as to say "There are red, blue, green and yellow". The pseudo-concept (colour) draws a boundary of language, the concept proper (red) draws a boundary in language. (Ibid., p. 12.)

One can see from these passages all the strengths and weaknesses of Wittgenstein's analysis at the same time. I have mentioned some of them; here I would just like to insist that Wittgenstein discerns a difference of levels in language but cannot speak of it very clearly.²² Concepts are articulated in language in complex manners which have only recently begun to be fathomed out by operational linguistics (see other contributions to this journal). Now, the

important thing is the organization of discourse around the predicates. That Wittgenstein had an inkling of the importance of this can be seen from a passage in his notebooks:

Nenne ich z.B. irgend einen Stab "A", eine Kugel "B", so kann ich von A sagen, es lehnt an der Wand, aber nicht von B. Hier macht sich die interne Natur von A und B bemerkbar. (...)
"Die Uhr *sitzt* auf dem Tisch" ist sinnlos! (Von Wright/Anscombe eds. 1979:70.)

In a curious mixture of metaphysics and linguistic analysis Wittgenstein shows once again a strong bias for nouns. He insists on the "internal nature" of objects. But first, it is not the nature of objects but the semantic features of nouns which are relevant here; and second, the verbs "lean" and "sit" are instrumental in that covert classification of nouns which we call a selection restriction.²³ This covert classification is also part and parcel of mathematicians' ordinary language (cf. Leal 1986) and the very small letters which in the first passage quoted on the last page indicate the domain of the variables (reduce the scopus of the variable x to numbers) are quite unnecessary, although they serve to show a malaise on the part of logicians which is very instructive. As a matter of fact, classification and thus individuation by means of variables depends upon a two-fold opposition. An a is first different from a b and from a c in that they are different letters of the same series; thus they potentially represent different individuals. On the other hand, an a is the same letter as an A , but they again differ in that the former is a smallcase letter (it belongs to the set a, b, c, \dots) whereas the latter is a capital (it belongs to the set A, B, C, \dots). This classification by shape is essential, as anyone will see who has read a book of mathematics. The virtuosity displayed by mathematicians in using different types, founts and alphabets is really as wonderful as it is to the point. And the point of it all is the APPREHENSION of objects.²⁴

Thus, the constitution of objects in a text must be seen as the result of the complex interplay of typically nominal procedures, like the "variable name", and typically verbal procedures, like the organization of a universe of discourse in which different kinds of objects may be talked of. The scare-quotes may now be left aside, because the operational sense of classification as a constitutive part of the APPREHENSION of objects has hopefully become clearer by now.

7. Conclusion

To sum up, Wittgenstein wanted to abolish the use of an expression like *x is an object*, and to substitute for it the syntactic rules that govern the use of a variable like *x* in a given context. In a similar vein, operational linguists are against trivial uses of words like *object* (and expressions like *to apprehend an object* or *to denote an object*). Now, Wittgenstein's notion of a formal concept was on the right track. The word *object* has to be understood operationally not objectually; behind objects one has to look for APPREHENSION; and a formal concept is not an ordinary concept but rather an operational plan or Piagetian schema. Another matter is the real use of the word in ordinary contexts; but that was not, as I said at the beginning, Wittgenstein's goal.

On the other hand, we most emphatically deny that syntactic rules of the kind envisioned by Wittgenstein, i.e. distributional rules, are good enough. We think rather that such rules are only superficial manifestations of much more interesting and profound principles and operations. Wittgenstein had a good intuition for some of these (and that even before the publication of de Saussure's *Cours de linguistique générale* which was only the beginning of scientific linguistics!); but he certainly could not make a theory out of it—and so he was led to think that talk about language was doomed to nonsense. Otherwise, he may have come to the idea that the whole of metaphysics could be enlight-

ened by the discovery of the operations of the mind which underlie the secular search for "first principles"

Thus, the fact that both the calculi Wittgenstein wanted to illuminate and the symbolisms he proposed to do so show the same active principles that operational linguistics has taught us to discern in the most diverse natural languages of the world opens the way to a collaboration between philosophers and linguists that might bring us, in philosophy, beyond the unfertile contest between "constructionists" and "ordinary language" analysts. Such a collaboration should be inscribed in a non-eurocentric effort to study not only language but languages; for without languages, language can only be a *Hirngespinnat*.

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APPENDIX (On propositional letters as analogues of abstract nouns)

First, I must insist that formal languages are metalinguistic in character. Not only do they presuppose the knowledge of a natural language but they are intended to schematize operations realized in natural languages. One of these operations is nominalization, which is not a matter of all-or-nothing but rather a more or less continuous progress from a statement (which may be true or false) to a proposition (which is a predicate apprehended as an object). When I say that *p* is similar or analogous to an abstract noun, I mean to establish first a parallel of techniques and second a difference in intention. It would be much more exact to say that *p*, *q*, etc. are the maximum schematization of possible degrees of nominalization.

Now, gradual nominalization is not identical with generalization (thus, the latter is present in a question, as opposed to a statement, but a question does not have any "nouniness" in itself) yet implies it. And again, generalization is not identical with universal quantification, although the mathematical tendency to extensionalism generally leads to confusion in this point. Thus, consider the usual procedure of mathematicians when proving a theorem. They say things like "Let *ABC* be a triangle..." or "Let *x* be a prime number..." These expressions, *ABC* and *x*, are real (unbound) variables. As Russell observed long ago, one has to distinguish between ϕx and $\phi \hat{x}$ in that the former is a statement (and therein the *x* a kind of deictic pronoun) whereas the latter is

not. According to ordinary extensionalist theorizing, only statements (like, say, ϕx) may be connected by means of the 'hook': $\phi x \supset \psi x$. People think of the 'hook' as a conjunction and believe to know that conjunctions connect statements. The core of the puzzle lies here: either is the 'hook' a conjunction or a verb—and therefore either are p and ϕx statements or else they are nouns.

The puzzle is a classical product of categorial thinking. Nominalization is gradual, even in the case of formal languages. (Formal languages are created by people but neither are people completely free to create whatever they like nor are their creations wholly arbitrary.) As a matter of fact, the English conjunction *if* does not connect statements at all, as is shown by the TAM restrictions of the sentences that may be conjoined by it. These restrictions are the beginning of a whole scale of nominality which culminates in the prototypical abstract nouns (cf. Iturrioz 1985). David Lewis and others have suggested that *if* contains a nonspecific quantification, i.e. a quantification which does not involve any particular "open place" of the predicate. From our standpoint, it is rather a matter of generalization (of desindividualization) than it is a matter of quantification. With the appearance of *if* an intensional discourse starts; and the same is in principle true of the 'hook'. But mathematical predicates are traditionally extensional and resist therefore an intensional analysis. Thus appears the interpretation of $\phi x \supset \psi x$ as $(x)(\phi x \supset \psi x)$, i.e. as implicitly quantified and thus extensional. This represents a distortion of the linguistic (and probably also of the psychological) facts, which may not do any harm as long as we safely remain within extensional theories. Anyway, it does not explain anything.

Now we are prepared to understand what I suggested above: if a sentence like $\phi x \supset \psi x$ can be interpreted by English *if it rains then it is wet*, then a sentence like $p \supset q$ should read rather like *rain implies humidity*. In natural language there is a subtle scale on which *if...then* and *implies* mark different positions and correspond to different kinds of text. Although the 'hook' remains one sign, there are here subtle differences, too. (Subtle and again gradual—after all, one can also say, colloquially, *if rain then humidity*.) But to observe them, we have to observe the system as such. Between predicate logic and propositional logic (the logic of predicates as objects) there is a difference of degree: the processes of nominalization and generalization started in the former come to fruition in the latter. Only the latter can express such general laws as *modus ponens* in an adequate way (i.e. in its full generality).

A further aspect is saturability. If you consider the prototypical—the full referential statement—in English, you will see that its predicate is a finite verb with a valency that is fully employed, i.e. obligatorily saturated. As soon as generalization enters the stage, obligatoriness begins to give way. With nominalization this process continues and a rigidization takes place which is the contrary of our starting point (or rather its mirrored image). Thus, infinitives and gerunds either do not accept saturation of "open places" or else they only accept a certain kind. When the process attains its end—with prototypical abstract nouns—a new scale appears in that saturability becomes fully, or almost fully, optional: it is the subdimension of ABSTRACTION within the dimension of APPREHENSION. This subdimension allows for a different

comparison between propositional logic and predicate logic. If you consider the development of predicate logic (the theory of apparent variables) out of propositional logic in *Principia mathematica*, you will easily observe the progress from $p \equiv q$ through $p \equiv_x q$ and $\phi x \equiv_x \psi x$ until $(x).\phi x \equiv \psi x$; in some other authors, the progress looks rather like this: $p \equiv q$ through $p_x \equiv q_x$ till $(x)(p_x \equiv q_x)$. Anyway, the whole process can be viewed either from the standpoint of progressive de-nominalization or from the standpoint of progressive saturation (although the real process is the other way round). In both cases we have generalization as a common factor.

FOOTNOTES

¹ See Seiler/Lehmann eds. 1982, Seiler/Stachowiak eds. 1982, Seiler 1986.

² Another aspect of the continuum is order, about which see Seiler 1986.

³ Although Wittgenstein wrote in German, and I shall also quote him mainly in German for the sake of precision, good style obliges me to talk as if he was referring to the equivalent English words. As a matter of fact, he often was, since the *Tractatus* is, to a considerable extent, an answer to Russell's work. Therefore, no real danger is involved.

⁴ This is 4.1272 quoted at length. Wittgenstein's original paragraphing has been slightly changed. Instead of the compound "Aleph-Null" the original has the corresponding Hebraic letter and an index.

⁵ Some readers may feel irritated by the apparent confusion in the expression "be represented by variables not by functions or classes", because the former seem to be symbols whereas the latter do not. This is not just a case of pre-Tarskian confusion of levels, but the point is rather that words like *object*, *function* and *class* are also metalinguistic.

⁶ Cf. 3.331-333.

⁷ Frege was also interested in similar distinctions, although he does not use these terms.

⁸ Instead of "facts", Wittgenstein spoke of "states of affairs" and he reserved the former term for "logical products" of "states of affairs" (cf. von Wright/Anscombe eds. 1979:130).

⁹ We have here again the problem of apparently incompatible expressions that we had pointed out in note 5; and the point is again that the word *number* is thought to be metalinguistic (as are the words *fact* and *complex* in the quoted passage). Incidentally, the "propositional variable" (*Satzvariable*) introduced in 3.313 is not the same as the "variable proposition" (*variabler Satz*) of 3.315; the former can be defined, grossly, as the class of all sentences which contain a given variable (and thus define that variable) whereas the latter is any given sentence which contains at least one variable.

¹⁰ Cf. "Die logischen Formen sind zahllos." (4.128); Wittgenstein's choice of words and emphasis suggests that he would not have thought it sensible to ask how many formal concepts there are (*zahllos* does not mean

the same as *unzähllich*, i.e. *abzählbar unendlich*; and there is an interesting parallelism in Wittgenstein between *unzähllich* and *unsinnig* on the one hand, and *zahllos* and *sinnlos* on the other hand).

11 For some ideas I refer the interested reader to Iturrioz/Leal 1986: 86ff.

12 "Von zwei Dingen zu sagen, sie seien identisch, ist ein Unsinn, und von Einem zu sagen, es sei identisch mit sich selbst, sagt gar nichts." (5.5303); in other words, the form "a = a" is a tautology and the form "a = b" is nonsense (cf. also 5.533-5.5352). There is an interesting difference in the way Frege and Wittgenstein look at symbols which I cannot here but recommend the reader to brood over: Why should the expression "a = b" look like nonsense?

13 Wittgenstein's obsession with one letter for one object (which we already remarked above) is manifested in a very puzzling manner in his introduction of the very curious symbol \bar{p} , which apparently covers both p and q , r , ... (cf. 6; see also 5.501). It is at bottom connected with his idea that the "variable name" denotes the formal concept of an object; how could he justify the use of several letters for one concept—even if it was a pseudo-concept? He even tried once to avoid this and signally failed (cf. 4.0411).

14 A recent essay by Kit Fine on so-called "arbitrary objects" tries to construct a predicate calculus without quantifiers. I cannot judge yet whether this attempt vindicates traditional usage in a way that both throws light on the linguistic question and satisfies logicians' purism.

15 The truth-table method was introduced in the sections 4.3 and 4.4 of the *Tractatus*.

16 Incidentally, there is also a difference in the degree of descriptivity (again, a UNITYP concept). Taking a hint from a later passage (6.01), an equivalent symbol would be $[x, O^2x]^2(a)$ which expresses the application of the recursive operation. This suggests the following scale (where ">" means "is more descriptive than"):

$$[x, O^2x]^2(a) > [a, x, O^2x] > x$$

The first expression is like an abstract noun with an oblique NP; this NP is than "incorporated" to the abstract noun in the second expression; and the third is just a "label" (for all this see Seiler 1975 and Iturrioz/Leal 1986:14).

17 On condensation see Iturrioz 1984 and forthcoming.

18 As usual, the discussion has been thoroughly prescriptive and damnable ("you should not..."), no attempt at understanding or explanation being made.

19 For Dyrbal see the Appendix on "Dyrbal Logic" in Dixon 1972. His translation of the relevant verb makes it appear as "modal": $p \vee q$ would mean something like "it might be that p , it might be that q " (there are similar expressions even in English). This is interesting in view of two other facts: that Western "modal logic" was born as an effort to understand the logical connective " \supset "; and that $p \supset q$ was easily "translated" by $\sim p \vee q$. The fact that a translation of both " \supset " and " \vee " in terms

of " \supset " implies a conversion of the whole truth table by means of negation—i.e. $(p \supset q) \equiv \sim(p \cdot \sim q)$ and $(p \vee q) \equiv \sim(\sim p \cdot \sim q)$ —makes one speculate whether the three connectives could not be ordered along a continuum of increasing "modality", something like

$$p \supset q > p \vee q > p \cdot q$$

with ">" meaning "more modal than". Now, the fact that implication is a form of quantification (Iturrioz) and generally von Wright's rapprochement of modal operators and quantifiers (nowadays made clearer by "possible worlds semantics") must be kept in mind when trying to understand propositional variables as analogues of abstract nouns. A last brief indication is that one occasionally finds textbook treatments which take advantage of the operation of saturation (cf. Seiler 1986:30ff.) by going from p to p_x , which would mean that the propositional variables are just unsaturated predicate variables (see below, esp. Appendix).

20 But compare the following passage from his "Notes on Logic" of 1913: "Signs of the form ' $p \vee \sim p$ ' are senseless, but not the proposition ' $(p) \cdot p \vee \sim p$ '." (Von Wright/Anscombe eds. 1979:104.) The universal quantifier represents a "semantic ascent" (Quine), so that the new proposition may occupy a place in an appropriate—higher—universe of discourse.

21 Cf. Iturrioz 1984 and forthcoming. It would be interesting to know how many times a similar thought has been repeated down the centuries—from Aristotle to Port-Royal and from Port-Royal to Chomsky and Lees—before a real analysis was undertaken.

22 I cannot go into still other passages in which Wittgenstein comes back to the differences between substantives and adjectives and especially to the levels of language which are manifest in, say, the way we use words like *man* or *chair* as against *red* (Ambrose 1979:125ff.); but it is clear that only the usual defect of philosophers of language, viz. the asystematicity of their linguistic observations (cf. Iturrioz/Leal 1986:85), prevented his doubtless sharp eye to come to real insight.

23 Wittgenstein would have been surprised to know that an explicit classification by verbs is part of the structure of many languages and of the APPREHENSION of objects that becomes thereby possible (cf. Seiler 1986: 77ff. and references there). On the other hand, a charitable reading of "the internal nature of objects" would be "the formal/operational characteristics of the objects as constituted by any given linguistic technique". That "internal" is almost a synonym for "formal" may be inferred from the use of both terms in the *Tractatus*; and that "formal" has to be spelled out operationally is part of what I have been trying to show in this paper; thus, "object" would be a strictly linguistic (i.e. linguistically defined) term and no admixture of metaphysics should be necessary.

24 I would like to add here that variables like x are y not "temporary classifiers" (cf. Serzisko 1982) because they do not constitute predicates—or at least not different predicates. The concept of a "temporary" classifier may, however, be applied to other kinds of variables, e.g. to Dedekind's A_1 and A_2 , because the indexes are predicates of order (cf. Leal 1986).

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